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Two-Loop $\mathcal{O}(\alpha_s G_F m_t^2)$ Correction to the $H \to b\bar{b}$ Decay Rate

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Abstract

We present the two-loop $\mathcal{O}(\alpha_s G_F m_t^2)$ correction to the $b\bar{b}$ decay rate of the Standard-Model Higgs boson, assuming that the t quark is much heavier than the Higgs boson. Apart from the universal correction connected with the renormalizations of the wave function and the vacuum expectation value of the Higgs field, this involves vertex corrections specific to the presence of beauty in the final state. We calculate the latter by means of a low-energy theorem. All would-be mass singularities related to the b quark can be absorbed into the running Higgs-bottom Yukawa coupling. It turns out that the total $\mathcal{O}(\alpha_s G_F m_t^2)$ correction screens the leading high-m_t behaviour of the one-loop result by 71% to 75% for M_H between 60 GeV and $2M_W$.

1 Introduction

One of the great puzzles of elementary particle physics today is whether nature makes use of the Higgs mechanism of spontaneous symmetry breaking to generate the observed particle masses. The Higgs boson, H, is the missing link sought to verify this concept in the Standard Model. The failure of experiments at LEP 1 and SLC to detect the decay $Z \to f\bar{f}H$ has ruled out the mass range $M_H \leq 63.8$ GeV at the 95% confidence level [1]. At the other extreme, unitarity arguments in intermediate-boson scattering at high energies [2] and considerations concerning the range of validity of perturbation theory [3] establish an upper bound on M_H at $\left(8\pi\sqrt{2}/3G_F\right)^{1/2} \approx 1$ TeV in a weakly interacting Standard Model.

A Higgs boson with $M_H \lesssim 135$ GeV decays dominantly to $b\bar{b}$ pairs [4]. This decay mode will be of prime importance for Higgs-boson searches at LEP 2 [5], the Tevatron [6]—or a possible 4-TeV upgrade thereof [7]—, and the next e^+e^- linear collider [8]. Techniques for the measurement of the $H \to b\bar{b}$ branching fraction at a $\sqrt{s} = 500$ GeV e^+e^- linear collider have been elaborated in Ref. [9].

The present knowledge of quantum corrections to the $H \to b\bar{b}$ decay rate has been reviewed very recently in Ref. [4]. The QCD corrections are most significant numerically. In $\mathcal{O}(\alpha_s)$, their full m_b dependence is known [10]. In $\mathcal{O}(\alpha_s^2)$, the first [11] and second [12] terms of the expansion in m_b^2/M_H^2 have been found. The large logarithmic corrections of the forms $\alpha_s^n \ln^m(M_H^2/m_b^2)$ and $\alpha_s^n m_b^2/M_H^2 \ln^m(M_H^2/m_b^2)$, with $n \geq m$ and $n, m = 1, 2$, which are present when the on-shell scheme of quark mass renormalization is employed, may be succinctly absorbed into the running b-quark mass of the $\overline{\mathrm{MS}}$ scheme evaluated at scale M_H. In this way, these logarithms are resummed to all orders and the perturbation series converges more rapidly. The residual terms are perturbatively small. The

theoretical uncertainty related to the lack of knowledge of the nonlogarithmic $\mathcal{O}(\alpha_s^3)$ term is likely to be irrelevant for practical purposes [13]. The QCD correction relative to the Born approximation implemented with the pole mass ranges between -53% and -63% for M_H between 60 GeV and $2M_W$ [4].

The full one-loop electroweak corrections to the $H \to b\bar{b}$ decay width are well established [14,15]. They consist of an electromagnetic and a weak part, which are separately finite and gauge independent. The electromagnetic part emerges from the $\mathcal{O}(\alpha_s)$ correction in the on-shell scheme [10] by substituting αQ_b^2 for $\alpha_s C_F$, where $Q_b = -1/3$ and $C_F = \left(N_c^2 - 1\right)/(2N_c)$, with $N_c = 3$. For $M_H \ll 2M_W$, the weak part is well approximated by [15]

$$\Delta_{\text{weak}} = \frac{G_F}{8\pi^2\sqrt{2}} \left\{ m_t^2 + M_W^2 \left(\frac{3}{s_w^2} \ln c_w^2 - 5 \right) + M_Z^2 \left[\frac{1}{2} - 3\left(1 - 4s_w^2 |Q_b|\right)^2 \right] \right\}, \qquad (1)$$

where $c_w^2 = 1 - s_w^2 = M_W^2/M_Z^2$. Here it is understood that the Born result is expressed in terms of the Fermi constant [16],

$$\Gamma_0\left(H \to b\bar{b}\right) = \frac{N_c G_F M_H m_b^2}{4\pi\sqrt{2}} \left(1 - \frac{4m_b^2}{M_H^2}\right)^{3/2}. \qquad (2)$$

Equation (1) has been obtained by putting $M_H = m_f = 0$ $(f \neq t)$ in the expression for the full one-loop weak correction [15]. It provides a very good approximation up to $M_H \approx 70$ GeV, the relative deviation from the full weak correction being less than 15%. In view of evidence for a heavy t quark, with $m_t = (174 \pm 16)$ GeV [17], the first term of Eq. (1) is particularly important. It consists of a universal part, which contributes to all fermionic Higgs-boson decays, and a non-universal part, which is specific to $b\bar{b}$ production. The universal part arises from the renormalizations of the wave function and the vacuum expectation value of the Higgs field, while the non-universal part is due to the $b\bar{b}H$ vertex correction and the b-quark wave-function renormalization. At one loop, there is a large

cancellation between the universal and non-universal parts, their sum being seven times smaller than the universal part alone.

The gluon correction to the shift in $\Gamma\left(H \to f\bar{f}\right)$ due to a doublet of novel quarks with arbitrary masses has been evaluated recently in Ref. [18]. The QCD correction to the $\mathcal{O}(G_F m_t^2)$ universal term emerges as a special case from this result [19]. It screens the one-loop term in such a way that the corrected term is given by $2\delta_{\mathrm{u}}$, where

$$\delta_{\mathrm{u}} = x_t \left[\frac{7}{2} - \frac{3}{2}\left(\zeta(2) + \frac{3}{2}\right) C_F \frac{\alpha_s}{\pi}\right], \tag{3}$$

with $x_t = \left(G_F m_t^2/8\pi^2\sqrt{2}\right)$. This correction constitutes the full $\mathcal{O}(\alpha_s G_F m_t^2)$ contribution to all fermionic decay widths of a Higgs boson with $M_H < 2m_t$, except for the $b\bar{b}$ channel. In the latter case, one needs to include the gluon correction to the $\mathcal{O}(G_F m_t^2)$ non-universal term. The evaluation of this correction is the subject of this article. For this purpose, we shall take advantage of a low-energy theorem [20,21,22]. Our final result disagrees with a recent analysis [23]. We shall pin down the error in Ref. [23].

This paper is organized as follows. In Sect. 2, we shall describe the low-energy theorem, which we shall apply in Sect. 3 to derive effective Lagrangians for the $b\bar{b}H$ and $b\bar{b}gH$ interactions to $\mathcal{O}(\alpha_s G_F m_t^2)$. In Sect. 4, we shall use these effective Lagrangians to evaluate the $\mathcal{O}(\alpha_s G_F m_t^2)$ correction to $\Gamma\left(H \to b\bar{b}\right)$. We also present a master formula for $\Gamma\left(H \to b\bar{b}\right)$, which includes all known corrections. Our conclusions are summarized in Sect. 5.

2 Low-energy theorem

Low-energy theorems for Higgs-boson interactions have been studied in great detail in the literature [20,21,22]. These theorems relate the amplitudes of two processes which differ by the insertion of an external Higgs-boson line carrying zero momentum. They provide

a convenient tool for estimating the properties of a Higgs boson that is light as compared to the loop particles. For instance, they have been applied to derive low-M_H effective Lagrangians for the $\gamma\gamma H$ and ggH interactions at one [21] and two loops [22].

These low-energy theorems may be derived by observing that the interactions of the Higgs boson with the massive particles in the Standard Model emerge from their mass terms by substituting $m_i \to m_i(1 + H/v)$, where m_i is the mass of the respective particle, H is the Higgs field, and v is the Higgs vacuum expectation value. On the other hand, a Higgs boson with zero momentum is represented by a constant field, since $i\partial_\mu H = [P_\mu, H] = 0$, where P_μ is the four-momentum operator. This immediately implies that a zero-momentum Higgs boson may be attached to an amplitude, $\mathcal{M}(A \to B)$, by carrying out the operation

$$\lim_{p_H \to 0} \mathcal{M}(A \to B + H) = \frac{1}{v} \sum_i m_i \frac{\partial}{\partial m_i} \mathcal{M}(A \to B), \tag{4}$$

where i runs over all massive particles involved in the transition $A \to B$. Here it is understood that the differential operator does not act on the m_i appearing in coupling constants, since this would generate tree-level four-point interactions involving one Higgs-boson line, which are absent in the Standard Model. Special care must be exercised when this low-energy theorem is to be applied beyond leading order. Then it must be formulated for the bare quantities of the theory. The renormalization is performed after the left-hand side of Eq. (4) has been evaluated.

3 Effective Lagrangians

We now turn to the $\mathcal{O}(\alpha_s G_F m_t^2)$ non-universal correction to $\Gamma\left(H \to b\bar{b}\right)$. Prior to performing the actual calculation, we outline the core of the procedure. Inspection of the one-loop weak correction to $\Gamma\left(H \to b\bar{b}\right)$ [15] reveals that only diagrams involving virtual

t quarks and charged Higgs-Kibble ghosts, $\phi^\pm$, and without direct $\phi^+\phi^- H$ coupling contribute to the $\mathcal{O}(G_F m_t^2)$ term. Moreover, the masses of the $\phi^\pm$ scalars may be put to zero. After factoring out the tree-level $b\bar{b}H$ amplitude, also the b quark may be treated as massless, so that the t quark is the only massive particle left in the loops.

The low-energy theorem (4) provides an alternative method of deriving the $\mathcal{O}(G_F m_t^2)$ non-universal correction to the $b\bar{b}H$ coupling, which requires only the computation of two-point functions. In fact, we just need to compute the b-quark self-energy amplitude induced by t and $\phi^\pm$ in the same approximation as above [see Fig. 1(a)]. The desired result may then be extracted by differentiation with respect to the bare b- and t-quark masses and performing their renormalizations in the resulting expression. The b-quark wave-function renormalization cancels against the corresponding part of the $b\bar{b}H$-vertex counterterm. By including also the $\mathcal{O}(G_F m_t^2)$ universal correction, which originates in the renormalizations of the Higgs-boson wave function and vacuum expectation value, we can formulate a low-M_H effective Lagrangian for the $b\bar{b}H$ interaction valid to $\mathcal{O}(G_F m_t^2)$, where the t quark is integrated out.

These considerations remain valid when one gluon is attached to the quark line in all possible ways. Since the gluon can occur as a virtual or a real particle, at first sight, it seems that one has to deal with both $b\bar{b}H$ and $b\bar{b}gH$ effective Lagrangians. However, it is easy to see that the $b\bar{b}gH$ coupling does not receive a contribution in $\mathcal{O}(G_F m_t^2)$. This may be understood by observing that the one-loop $b\bar{b}g$ vertex correction, from which the $b\bar{b}gH$ amplitude may be constructed by means of the low-energy theorem (4), does not develop a term proportional to m_t^2 in the high-m_t limit. The latter point may be inferred from the analogous calculation of the $b\bar{b}\gamma$ vertex at one loop [24]. Proceeding along the same lines as above and exploiting knowledge of the $\mathcal{O}(\alpha_s G_F m_t^2)$ universal correction [see Eq. (3)], it is possible to extend the $b\bar{b}H$ effective Lagrangian to $\mathcal{O}(\alpha_s G_F m_t^2)$.

Finally, we may embed this effective Lagrangian in the usual QCD Lagrangian involving five quark flavours and perform perturbation theory in α_s. In the case of $\Gamma\left(H \to b\bar{b}\right)$, we shall then recover the known $\mathcal{O}(\alpha_s)$ [10] and $\mathcal{O}(\alpha_s^2)$ corrections [11,12] with the leading m_t-dependent terms being collected in an overall factor. Apart from the advantage of being implemented conveniently, our result will resum automatically reducible higher-order terms. By including also the electromagnetic and remaining weak corrections, we shall obtain the complete Standard-Model prediction.

In summary, our original problem reduces to the evaluation of the self-energy of an on-shell b quark up to $\mathcal{O}(\alpha_s G_F m_t^2)$ in the limit of vanishing b-quark and W-boson masses. The relevant one- and two-loop diagrams are depicted in Figs. 1(a) and (b), respectively. As usual, we shall use dimensional regularization with anticommuting γ_5. Notice that we have not included the reducible two-loop diagrams in Fig. 1(b), since the one-loop gluon-exchange subdiagram vanishes, due to the absence of a mass scale to carry its dimension. These diagrams as well as real-gluon emission will come in at a later stage through the QCD corrections that are to be evaluated from the effective Lagrangian keeping the b-quark mass finite. This will be done in the next section.

The bare amplitude characterizing the propagation of an on-shell b quark in the presence of quantum effects has the form

$$\mathcal{M}^0(b \to b) = \left[m_b^0\left(-1+\Sigma_S^0(p^2)\right) + \not{p}\left(\Sigma_V^0(p^2)+\gamma_5\Sigma_A^0(p^2)\right)\right]_{\not{p}=m_b^0}, \tag{5}$$

where S, V, and A label the scalar, vector, and axial-vector components of the b-quark self-energy, respectively, and the superscript 0 marks bare quantities. Using the Dirac equation and putting $m_b^0 = 0$ in the loop amplitudes, this becomes

$$\mathcal{M}^0(b \to b) = m_b^0(-1+\Sigma), \tag{6}$$

where $\Sigma = \Sigma_V^0(0) + \Sigma_S^0(0)$. Evaluating the Feynman diagrams in Figs. 1(a) and (b), we

obtain the $\mathcal{O}(G_F m_t^2)$ and $\mathcal{O}(\alpha_s G_F m_t^2)$ terms of $\Sigma = \Sigma_1 + \Sigma_2 + \cdots$,

$$\begin{aligned}
\Sigma_1 &= -x_t^0 \left(\frac{4\pi\mu^2}{(m_t^0)^2}\right)^\epsilon \Gamma(1+\epsilon)\left(\frac{3}{2\epsilon}+\frac{5}{4}+\mathcal{O}(\epsilon)\right), \\
\Sigma_2 &= -C_F \frac{\alpha_s}{\pi} x_t^0 \left(\frac{4\pi\mu^2}{(m_t^0)^2}\right)^{2\epsilon} \Gamma^2(1+\epsilon)\left(\frac{9}{8\epsilon^2}+\frac{9}{8\epsilon}+\mathcal{O}(1)\right),
\end{aligned} \tag{7}$$

where $n = 4 - 2\epsilon$ is the dimensionality of space-time, μ is the 't Hooft mass, Γ is Euler's gamma function, and $x_t^0 = \left(G_F (m_t^0)^2/8\pi^2\sqrt{2}\right)$. We recall that the m_t-independent QCD corrections do not contribute to the effective Lagrangian in the limit of vanishing b-quark mass. Equation (4) now tells us that

$$\begin{aligned}
\lim_{p_H\to 0} \mathcal{M}^0(b\to b+H) &= \frac{1}{v_0}\left(m_t^0\frac{\partial}{\partial m_t^0} + m_b^0 \frac{\partial}{\partial m_b^0}\right)\mathcal{M}^0(b\to b) \\
&= \frac{m_b^0}{v_0}\left(-1+\Sigma+m_t^0\frac{\partial\Sigma}{\partial m_t^0}\right).
\end{aligned} \tag{8}$$

Thus, we evaluate

$$\begin{aligned}
m_t^0\frac{\partial\Sigma_1}{\partial m_t^0} &= x_t^0 \left(\frac{4\pi\mu^2}{(m_t^0)^2}\right)^\epsilon \Gamma(1+\epsilon)\left(3+\frac{5}{2}\epsilon+\mathcal{O}(\epsilon^2)\right), \\
m_t^0\frac{\partial\Sigma_2}{\partial m_t^0} &= C_F \frac{\alpha_s}{\pi} x_t^0 \left(\frac{4\pi\mu^2}{(m_t^0)^2}\right)^{2\epsilon} \Gamma^2(1+\epsilon)\left(\frac{9}{2\epsilon}+\frac{9}{2}+\mathcal{O}(\epsilon)\right),
\end{aligned} \tag{9}$$

treating x_t^0 as a constant, since it receives its two powers of m_t^0 from the $t\bar{b}\phi^-$ and $b\bar{t}\phi^+$ couplings.

Next, we carry out the renormalization procedure. For this end, we substitute $m_q^0 = m_q + \delta m_q$ $(q = t, b)$, where m_q is the on-shell mass and δm_q is the appropriately defined counterterm. For $q = b$, we have $\delta m_b/m_b = \Sigma$, so that

$$\lim_{p_H\to 0} \mathcal{M}^0(b\to b+H) = \frac{m_b}{v_0}\left(-1+m_t^0\frac{\partial\Sigma}{\partial m_t^0}\right), \tag{10}$$

which is correct through $\mathcal{O}(\alpha_s G_F m_t^2)$. We observe that $m_t^0(\partial\Sigma_1/\partial m_t^0)$ is already finite in the physical limit, $\epsilon \to 0$, as it must because it constitutes the first term in the series

of leading high-m_t non-universal corrections to the $b\bar{b}H$ effective coupling. $m_t^0(\partial\Sigma_2/\partial m_t^0)$ will become finite when we also renormalize the t-quark mass. To $\mathcal{O}(\alpha_s)$, we have

$$\frac{\delta m_t}{m_t} = -C_F\frac{\alpha_s}{\pi}\left(\frac{4\pi\mu^2}{m_t^2}\right)^\epsilon \Gamma(1+\epsilon)\left(\frac{3}{4\epsilon}+1+\mathcal{O}(\epsilon)\right). \tag{11}$$

In fact, this yields an ultraviolet-finite result,

$$\delta_{\mathrm{nu}} \equiv -m_t^0\frac{\partial\Sigma}{\partial m_t^0} = x_t\left(-3+\frac{3}{4}C_F\frac{\alpha_s}{\pi}\right), \tag{12}$$

where x_t is defined below Eq. (3). Note that Eq. (12) is μ independent as it should, since we are working in the on-shell scheme. Equation (12) reproduces the well-known $\mathcal{O}(G_F m_t^2)$ non-universal correction to $\Gamma\left(H\to b\bar{b}\right)$ [15] as may be seen by comparing $2(\delta_{\mathrm{u}}+\delta_{\mathrm{nu}})$ with Eq. (1). Inserting Eq. (12) in Eq. (10), we obtain

$$\lim_{p_H\to 0}\mathcal{M}^0(b\to b+H) = -\frac{m_b}{v_0}(1+\delta_{\mathrm{nu}}). \tag{13}$$

We are now in the position to write down the low-M_H effective Lagrangian for the $b\bar{b}H$ interaction including the $\mathcal{O}(G_F m_t^2)$ and $\mathcal{O}(\alpha_s G_F m_t^2)$ non-universal corrections,

$$\mathcal{L}_{\mathrm{eff}} = -\frac{m_b}{v^0}\bar{b}bH^0(1+\delta_{\mathrm{nu}}). \tag{14}$$

Here we have represented the b quarks by their renormalized fields, anticipating the cancellation of the corresponding wave-function renormalizations by an appropriate piece in the $b\bar{b}H$ vertex counterterm [15]. We still need to include the universal corrections. They enter through the relation $H^0/v^0 = (H/v)(1+\delta_{\mathrm{u}})$, where $v = 2^{-1/4}G_F^{-1/2}$ and δ_{u} is given by Eq. (3). As a result, Eq. (14) becomes

$$\mathcal{L}_{\mathrm{eff}} = -2^{1/4}G_F^{1/2}m_b\bar{b}bH(1+\delta_{\mathrm{u}})(1+\delta_{\mathrm{nu}}). \tag{15}$$

4 Results

In the previous section, we have constructed a low-M_H effective Lagrangian for the $b\bar{b}H$ interaction in the Standard Model by integrating out the t quark. Using this Lagrangian, we can now compute $\Gamma\left(H \to b\bar{b}\right)$ including the $\mathcal{O}(G_F m_t^2)$ and $\mathcal{O}(\alpha_s G_F m_t^2)$ corrections. By accommodating also the strong, electromagnetic, and residual weak corrections, we obtain the full Standard-Model prediction, which we may write in a factorized form,

$$\Gamma\left(H \to b\bar{b}\right) = \Gamma_{\mathrm{QCD}}\left(H \to b\bar{b}\right)(1+\delta_{\mathrm{u}})^2(1+\delta_{\mathrm{nu}})^2(1+\Delta_{\mathrm{weak}} - x_t)\left(1+\frac{\alpha}{\pi}Q_b^2\delta_{\mathrm{QED}}\right), \quad (16)$$

where δ_{u} and δ_{nu} are listed in Eqs. (3,12), respectively, and Δ_{weak} and δ_{QED} may be found in Ref. [15]. A low-M_H approximation for Δ_{weak} is given by Eq. (1). $\Gamma_{\mathrm{QCD}}\left(H \to b\bar{b}\right)$ contains the tree-level result of Eq. (2) along with its QCD corrections, which we have reviewed in the Introduction. Adopting the on-shell definition of quark mass, the $\mathcal{O}(\alpha_s)$ result reads

$$\Gamma_{\mathrm{QCD}}\left(H \to b\bar{b}\right) = \Gamma_0\left(H \to b\bar{b}\right)\left(1 + C_F\frac{\alpha_s}{\pi}\delta_{\mathrm{QED}}\right). \quad (17)$$

For $m_b \ll M_H/2$, δ_{QED} may be expanded as [15]

$$\delta_{\mathrm{QED}} = -\frac{3}{2}\ln\frac{M_H^2}{m_b^2} + \frac{9}{4} + \mathcal{O}\left(\frac{m_b^2}{M_H^2}\ln\frac{M_H^2}{m_b^2}\right). \quad (18)$$

It is interesting to study how the m_t-dependent term in Eq. (1) is affected by QCD corrections. Toward this end, we consider the product

$$\begin{aligned}&\left(1 + C_F\frac{\alpha_s}{\pi}\delta_{\mathrm{QED}}\right)(1+\delta_{\mathrm{u}})^2(1+\delta_{\mathrm{nu}})^2\\ &= 1 + \frac{3}{2}C_F\frac{\alpha_s}{\pi}\left(-\ln\frac{M_H^2}{m_b^2} + \frac{3}{2}\right) + x_t\left[1 - 3C_F\frac{\alpha_s}{\pi}\left(\frac{1}{2}\ln\frac{M_H^2}{m_b^2} + \zeta(2) + \frac{1}{4}\right)\right] + \cdots, \quad (19)\end{aligned}$$

where the ellipsis represents terms of $\mathcal{O}(\alpha_s m_b^2/M_H^2 \ln(M_H^2/m_b^2))$, $\mathcal{O}(\alpha_s^2)$, and $\mathcal{O}(G_F^2 m_t^4)$. We recover the notion that, in Electroweak Physics, one-loop $\mathcal{O}(G_F m_t^2)$ terms get screened by their QCD corrections. In the present case, the screening effect is extraordinarily

strong. In fact, for $M_H = 60$ GeV ($2M_W$), the $\mathcal{O}(\alpha_s G_F m_t^2)$ correction compensates 71% (75%) of the $\mathcal{O}(G_F m_t^2)$ term. Here, we have employed $m_b = 4.72$ GeV [25] and evaluated $\alpha_s(\mu)$ at renormalization scale $\mu = M_H$. As a normalization point, we have used $\alpha_s(M_Z) = 0.118$ [26]. However, we should bear in mind that the $\mathcal{O}(G_F m_t^2)$ term is incidentally small due to the almost complete cancellation of the universal and non-universal contributions.

The presence of large logarithmic terms like $\alpha_s \ln(M_H^2/m_b^2)$ is, of course, detrimental for the speed of convergence of the QCD perturbation series. However, from the organization of Eq. (16) it is evident that these logarithms may be rendered harmless in the usual way, by introducing the running b-quark mass of the $\overline{\mathrm{MS}}$ scheme evaluated at scale M_H. The appropriate formula for $\Gamma_{\mathrm{QCD}}\left(H \to b\bar{b}\right)$ is listed in Eq. (21) of Ref. [5]. By expanding the weak correction factor in Eq. (16), $(1+\delta_{\mathrm{u}})^2(1+\delta_{\mathrm{nu}})^2(1+\Delta_{\mathrm{weak}} - x_t)$, we recover Eq. (1) along with the QCD correction factor that multiplies the m_t-dependent term of Eq. (1). The latter reads

$$\begin{aligned} 1 - 3(\zeta(2)+1)C_F\frac{\alpha_s}{\pi} &= 1 - 2\left(\frac{\pi}{3} + \frac{2}{\pi}\right)\alpha_s \\ &\approx 1 - 3.368\,\alpha_s. \end{aligned} \tag{20}$$

In this way of presenting our result, the screening effect amounts to −42% (−37%) for $M_H = 60$ GeV ($2M_W$).

At this point, we should compare our analysis of $\Gamma\left(H \to b\bar{b}\right)$ with the result of a recent work [23]. According to Eq. (12) in Ref. [23], the $\mathcal{O}(G_F m_t^2)$ term gets dressed by the factor

$$K = 1 + \frac{\alpha_s}{\pi}\left(-22 - \frac{2}{3}\pi^2 + 12\ln\frac{M_H^2}{m_b^2}\right), \tag{21}$$

which has to be contrasted with the square bracket in our Eq. (19). We observe that the expression in Eq. (21) is significantly larger than our K factor, the ratio of the two

being 8.0 (11.8) at $M_H = 60$ GeV ($2M_W$). In other words, the authors of Ref. [23] find an enhancement of the $\mathcal{O}(G_F m_t^2)$ term by 130% (194%), whereas we find a reduction by 71% (75%). Comparing Eqs. (19,21), we can trace the source of this discrepancy. In fact, up to terms of $\mathcal{O}(\alpha_s^2 G_F m_t^2)$, we can rewrite Eq. (21) as $K x_t = 2\delta_{\rm u} + 2\delta_{\rm nu}[1 + C_F(\alpha_s/\pi)\delta_{\rm QED}]$, i.e., the interference of the $\mathcal{O}(G_F m_t^2)$ universal term and the $\mathcal{O}(\alpha_s)$ term, $2\delta_{\rm u} C_F(\alpha_s/\pi)\delta_{\rm QED}$, is missing in Eq. (21).

5 Conclusions

We computed the two-loop $\mathcal{O}(\alpha_s G_F m_t^2)$ non-universal correction to $\Gamma\left(H \to b\bar{b}\right)$, which arises from the $b\bar{b}H$ vertex and the b-quark wave function, by means of a low-energy theorem. Combining this result with the universal correction in the same order, which is due to the renormalizations of the Higgs-boson wave function and vacuum expectation value and contributes to any Higgs-boson decay to fermions or intermediate bosons, we obtained the full $\mathcal{O}(\alpha_s G_F m_t^2)$ correction to $\Gamma\left(H \to b\bar{b}\right)$. This correction screens the positive $\mathcal{O}(G_F m_t^2)$ term by 71% to 75% for M_H between 60 GeV and $2M_W$. As a consequence, the sensitivity of $\Gamma\left(H \to b\bar{b}\right)$ to the top quark, which, at one loop, is already seven times weaker than in the case of the other fermionic decay modes, is practically quenched. We presented a master formula for $\Gamma\left(H \to b\bar{b}\right)$, which makes full use of the present knowledge of radiative corrections to this quantity. We conclude that the residual theoretical uncertainty due to unknown higher-order corrections is likely to be negligible as compared to the envisaged experimental error.

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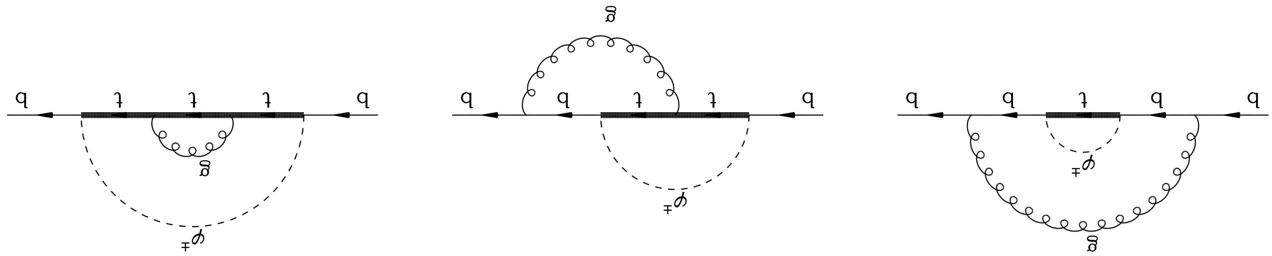

(a)

(b)

FIGURE CAPTION

Figure 1: Feynman diagrams pertinent to the b-quark self-energy in (a) $\mathcal{O}(G_F m_t^2)$ and (b) $\mathcal{O}(\alpha_s G_F m_t^2)$.